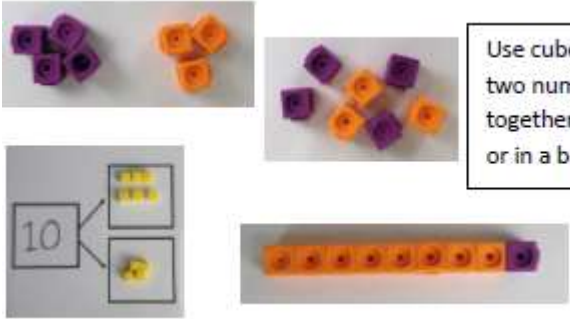
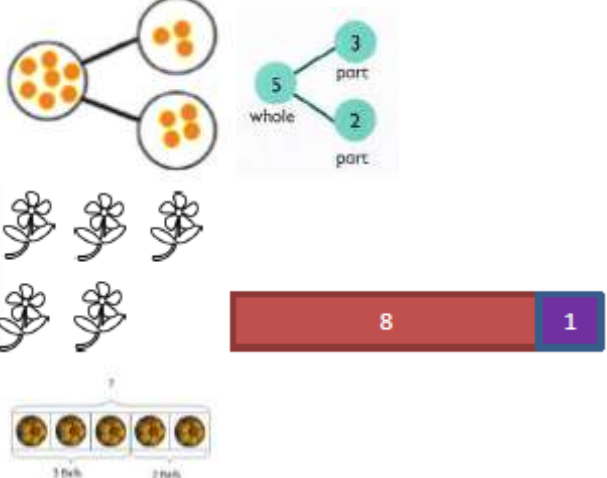
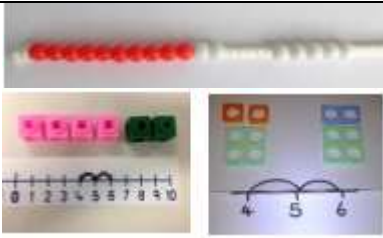
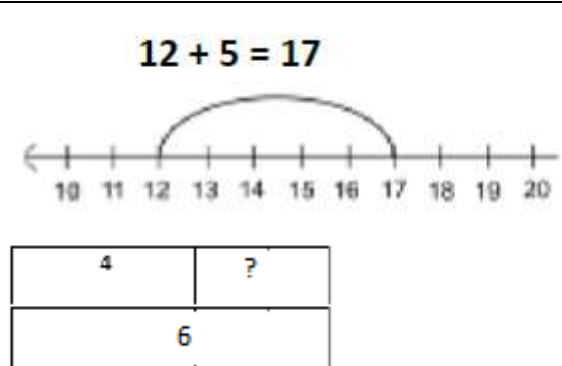
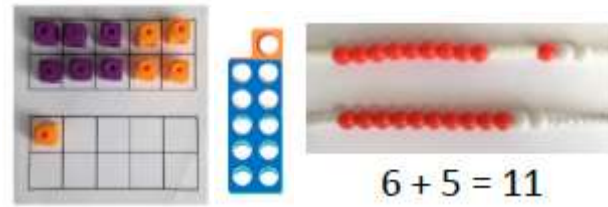
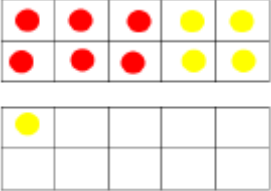
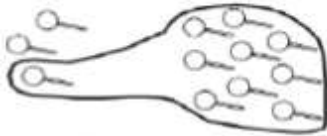
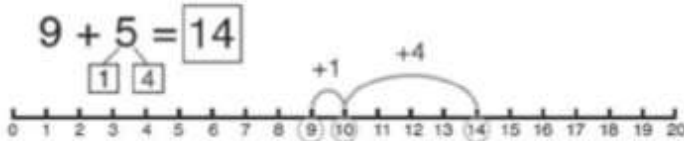

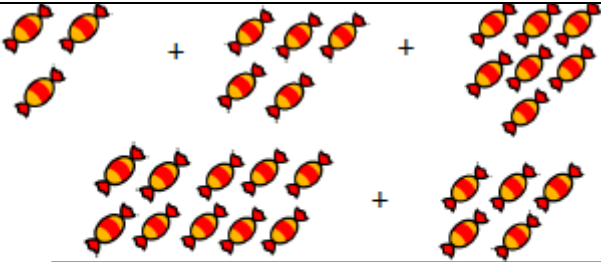
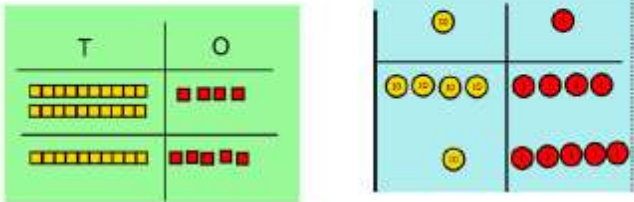
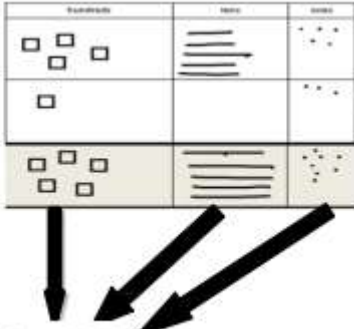


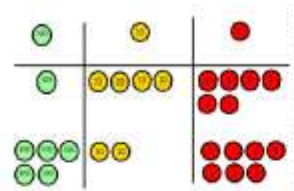
# Addition

Objective	Concrete	Pictorial	Abstract
Combining two parts to make a whole	 <p>Use cubes to add two numbers together as a group or in a bar.</p>		$4 + 3 = 7$  Four is a part, three is a part and the whole is seven.  Addend + addend = sum  $7 = 4 + 3$
Counting on from the biggest number		$12 + 5 = 17$ 	$12 + 5 = 17$  $17 = 12 + 5$  $4 + 2 = 6$  $6 = 4 + 2$
Regrouping to make 10	$6 + 5$  $6 + 5 = 11$		$9 + 5 = 14$ $9 + 1 + 4 = 14$  $7 + 4 = 11$ $7 + 3 + 1 = 11$ If I am 7, how many more do I need to make 10? How many more do I need to add on now?  Can I use my number bonds?  Can I regroup?

		 $3 + 9 =$ $9 + 5 = 14$ 	
Adding three single digits	$4 + 7 + 6 = 17$ <p>Put 4 and 6 together to make 10. Add on 7.</p> 	 <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p>	$\begin{array}{r} 4 + 7 + 6 = 10 + 7 \\ \quad \quad \quad 10 \\ = 17 \end{array}$
Column addition - Partitioning to add without regrouping	$24 + 15 =$ <p>Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p> 	 $455 + 103 = 558$	$24 + 15 = ?$ $20 + 10 = 30$ $4 + 5 = 9$ $30 + 9 = 39$ $\begin{array}{r} 24 \\ + 15 \\ \hline 39 \end{array}$

Column addition –  
with regrouping  
(Year 2+)

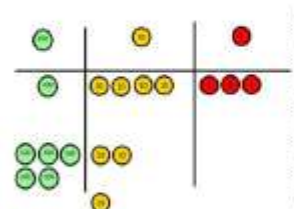
Make both numbers on a place value grid.



146

+ 527

Add up the ones and  
exchange 10 ones for one  
10.



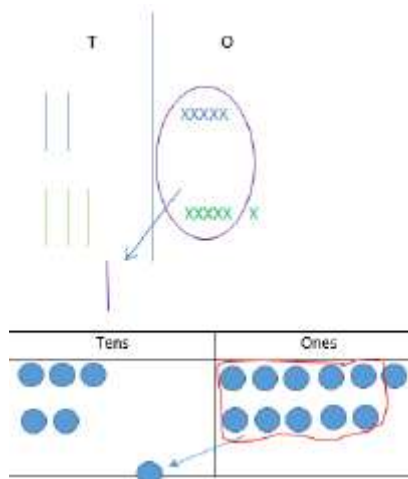
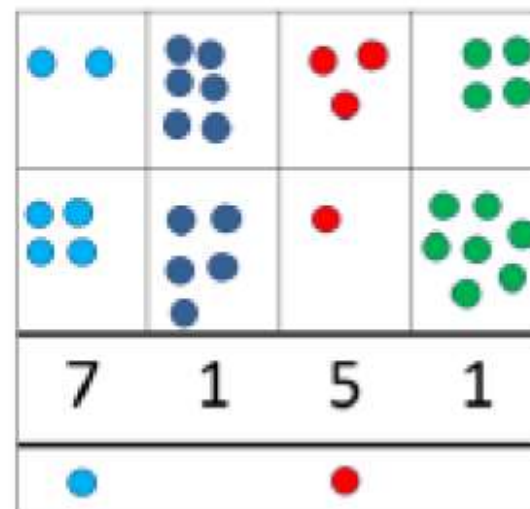
146

+ 527

Add up the rest of the columns, exchanging the 10 counters from  
one column for the next place value column until every column has  
been added.

This can also be done with Base 10 to help children clearly see that  
10 ones equal 1 ten and 10 tens equal 100.

Children can draw a pictorial representation of the columns and place  
value counters to further support their learning and understanding.



Looking for ways to make 10

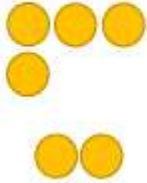
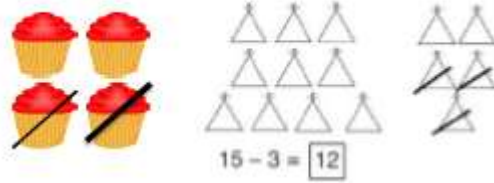


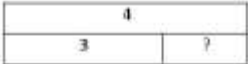
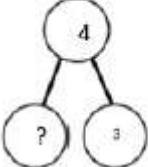



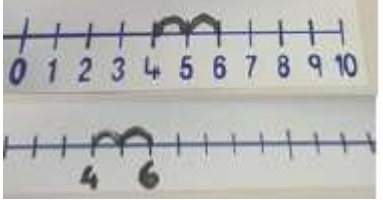
$$\begin{array}{r} 36 + 25 = \\ \begin{array}{cc} 1 & 5 \end{array} \end{array}$$

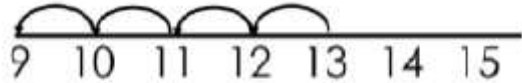
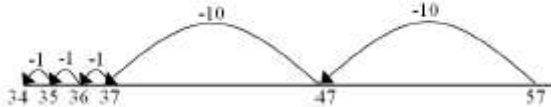

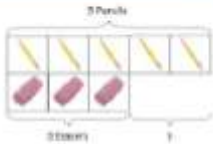
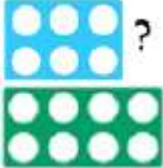
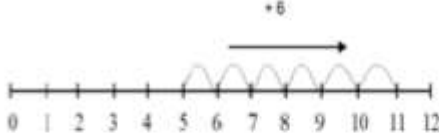
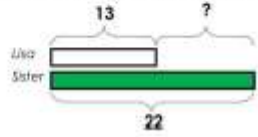
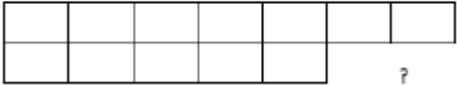
30 + 20 = 50  
5 + 5 = 10  
50 + 10 + 1 = 61

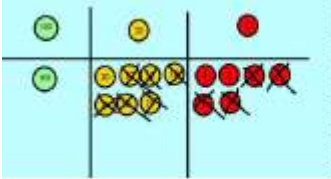
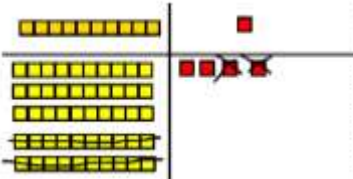
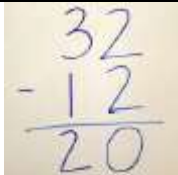
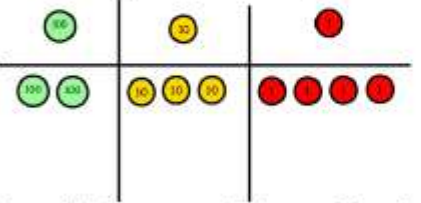
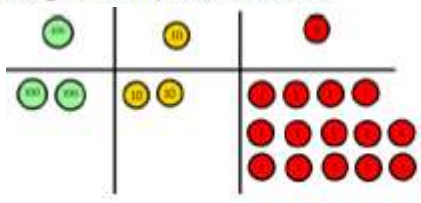
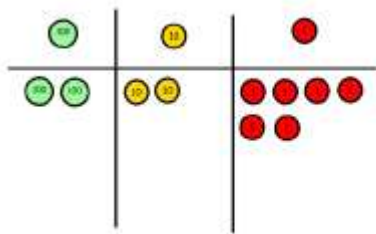
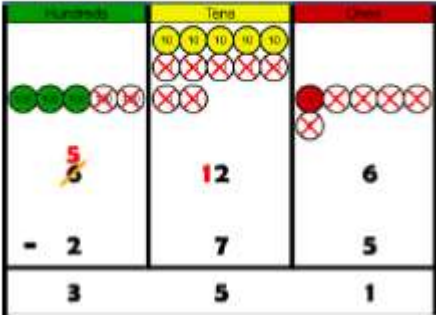
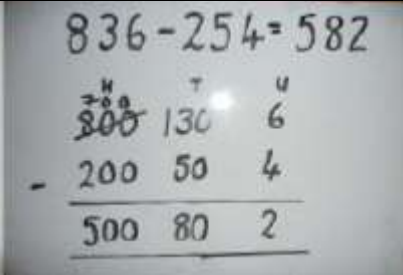

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

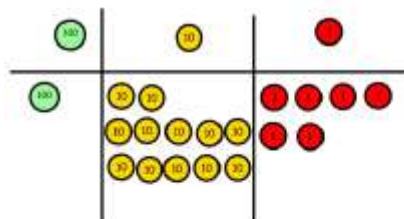
# Subtraction

Objective	Concrete	Pictorial	Abstract
Taking away ones	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  $6 - 2 = 4$	<p>Cross out drawn objects to show what has been taken away.</p>  <p>Children to draw the concrete resources they are using and cross out.</p>  <p>Use of the bar model:</p>   	$4 - 3 =$ 
Counting back	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p> $13 - 4$ 		 <p>Six subtract four is two.</p>

		<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Minuend – subtrahend = difference</p>
Find the difference	  	 <p>Count on to find the difference.</p> <p><b>Comparison Bar Models</b></p> <p>Draw bars to find the difference between 2 numbers.</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p>  <p>Children to draw the cubes/other concrete objects which they have used</p> <p>XXXXXXXX XXXXXX</p> <p>Use of the bar model</p> 	<p>The difference between 8 and 6 is 2.</p> <p>Children to also explore why <math>9 - 7 = 8 - 6</math> (the difference of each digit has changed by 1).</p>

<p>Column subtraction without regrouping</p>	 <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}</math> </div> <div> <p>Calculations</p> <math display="block">176 - 64 = 112</math> </div> </div>	 <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 542 \\ - 22 \\ \hline 32 \end{array}</math> </div> <div> <p>Calculations</p> <math display="block">542 - 22 = 32</math> </div> </div>	
<p>Column subtraction with regrouping (Year 2+)</p>	<p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> <p>Make the larger number with the place value counters</p>  <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}</math> </div> <div> <p>Calculations</p> <math display="block">234 - 88</math> </div> </div> <p>Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p>  <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}</math> </div> <div> <p>Calculations</p> <math display="block">234 - 88</math> </div> </div> <p>Now I can subtract my ones.</p>  <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}</math> </div> <div> <p>Calculations</p> <math display="block">234 - 88</math> </div> </div> <p>Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.</p>	<p>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.</p> 	 

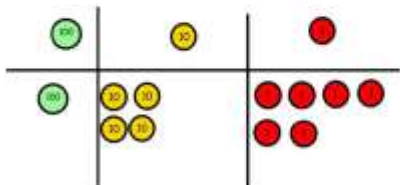




Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Now I can take away eight tens and complete my subtraction



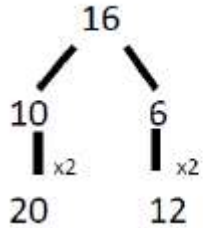



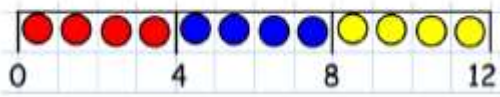


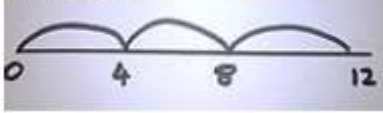


Calculations

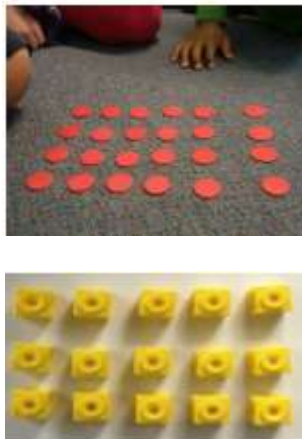
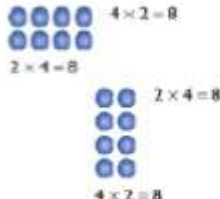



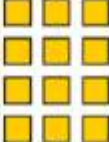



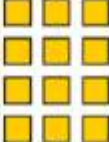
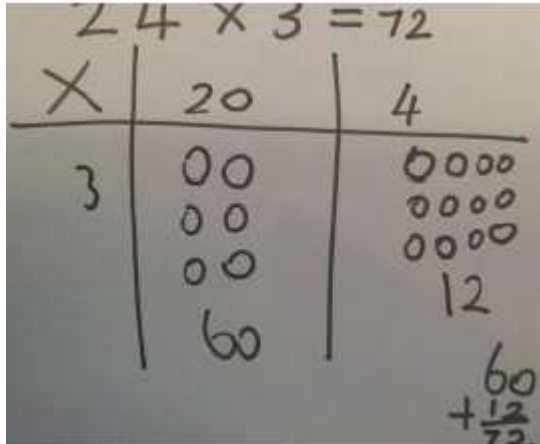



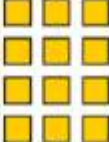
$$\begin{array}{r} 234 \\ - 88 \\ \hline 146 \end{array}$$

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

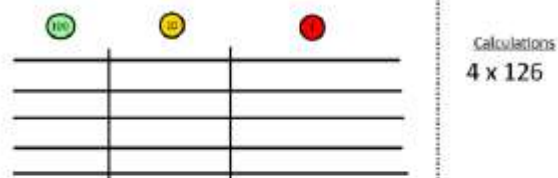
# Multiplication

Objective	Concrete	Pictorial	Abstract
Doubling	 <p>double 4 is 8 <math>4 \times 2 = 8</math></p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Repeated addition	<p><b>Repeated grouping/repeated addition</b> (does not have to be restricted to cubes) <math>3 \times 4</math> or 3 lots of 4</p>  	<p>Children to represent the practical resources in a picture e.g.</p> <p>XX XX XX XX XX XX</p> <p>Use of a bar model for a more structured method</p>  <p>Represent this pictorially alongside a number line e.g:</p>   <p><math>5 + 5 + 5 = 15</math></p> <p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>  <p>2 add 2 add 2 equals 6</p>	<p><math>3 \times 4</math></p> <p><math>4 + 4 + 4</math></p> <p>Abstract number line <math>3 \times 4 = 12</math></p>  <p>Factor x factor = product</p>

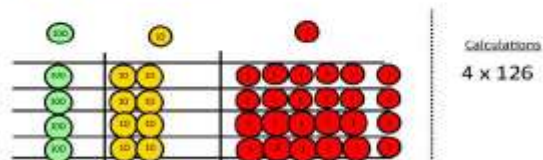


Arrays (commutative )		 <p>Draw arrays in different rotations to find commutative multiplication sentences.</p>	$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$																											
Grid method (Year 3+)	<p>Show the link with arrays to first introduce the grid method.</p> <table data-bbox="253 601 714 772"><tr><td>x</td><td>10</td><td>3</td></tr><tr><td>4</td><td></td><td></td></tr></table> <p>4 rows of 10 4 rows of 3</p> <p>Move on to using Base 10 to move towards a more compact method.</p> <table data-bbox="259 868 712 1098"><tr><td>x</td><td>T</td><td>U</td></tr><tr><td></td><td></td><td></td></tr></table> <p>4 rows of 13</p>	x	10	3	4			x	T	U					<table data-bbox="1556 566 2114 732"><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> $210 + 35 = 245$ <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table data-bbox="1630 992 2027 1251"><tr><td></td><td>10</td><td>8</td></tr><tr><td>10</td><td>100</td><td>80</td></tr><tr><td>3</td><td>30</td><td>24</td></tr></table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24
x	10	3																												
4																														
x	T	U																												
																														
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7	210	35																												
	10	8																												
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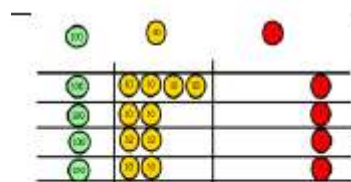
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.

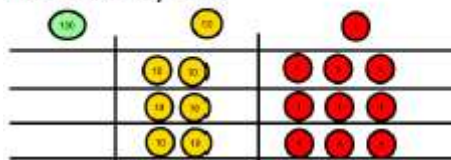


Then you have your answer.



Column  
multiplication  
(Year 4+)

Make 23, 3 times. See how many ones, then how many tens



Children to represent the counters in a pictorial way





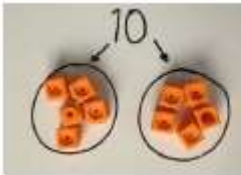

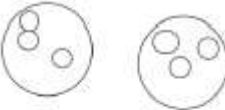


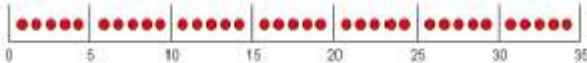

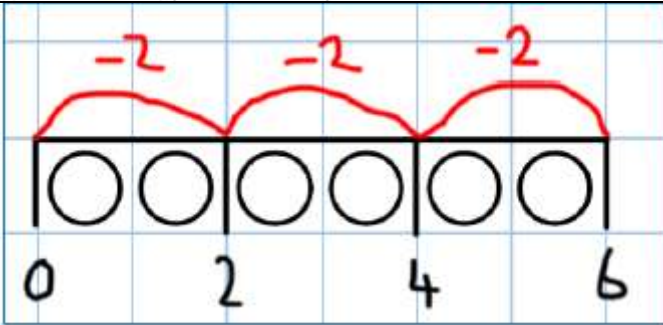
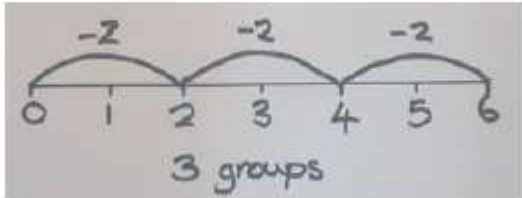
Children to record what it is they are doing to show understanding

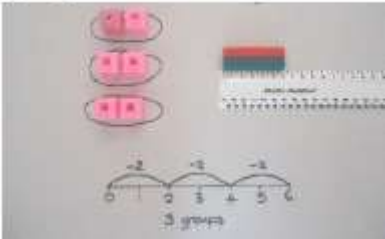
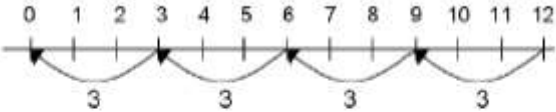
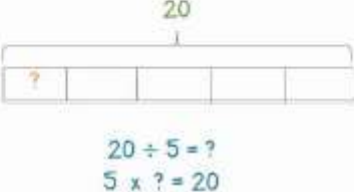
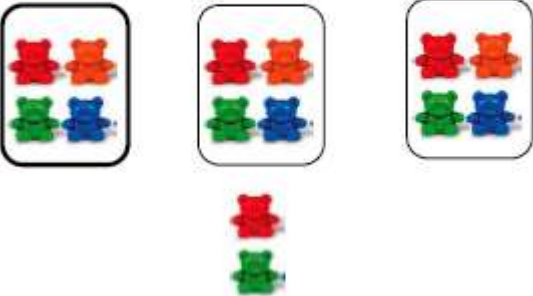



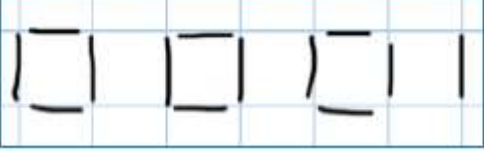
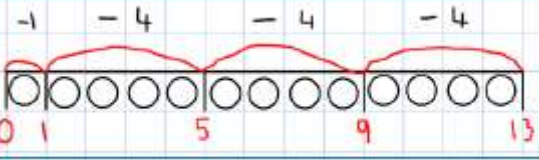
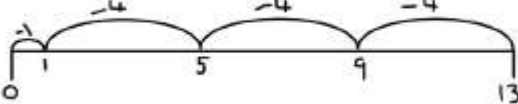
$$\begin{array}{l} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \end{array} \quad \begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$



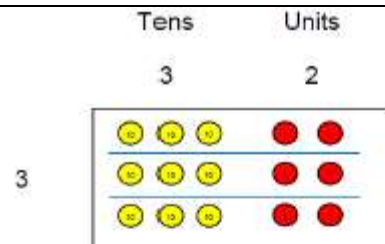
# Division

Objective	Concrete	Pictorial	Abstract		
Sharing objects into groups	<div></div> <div></div> <div></div> <div>I have 10 cubes, can you share them equally in 2 groups?</div>	<div>Children use pictures or shapes to share quantities.</div> <div></div> <div><math>8 \div 2 = 4</math></div> <div></div> <div>This can also be done in a bar so all 4 operations have a similar structure:</div> <div></div>	<div><math>6 \div 2 = 3</math></div> <div>What's the calculation?</div> <div><table><tr><td>3</td><td>3</td></tr></table></div>	3	3
3	3				
Division as grouping	<div>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</div> <div></div> <div><math>96 \div 3 = 32</math></div> <div></div> <div></div>		<div>Abstract number line</div> <div></div> <div><math>6 \div 2 = 3</math></div>		

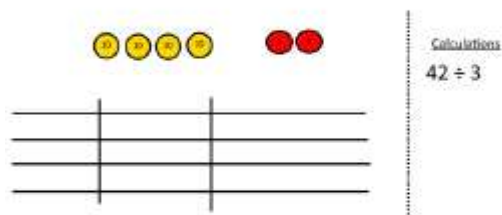
	<p><math>6 \div 2</math></p> 	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p> 	
<p>Division with a remainder (Year 2+)</p>	<p><math>14 \div 3 =</math></p> <p>Divide objects between groups and see how much is left over</p>  <p>Use of lollipop sticks to form wholes</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>   	<p><math>29 \div 8 = 3 \text{ REMAINDER } 5</math></p> <p>↑     ↑     ↑     ↑ dividend   divisor   quotient   remainder</p> 



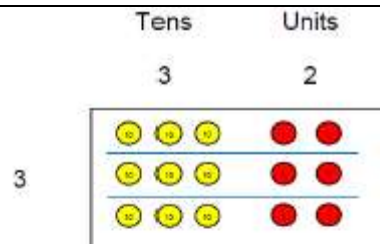
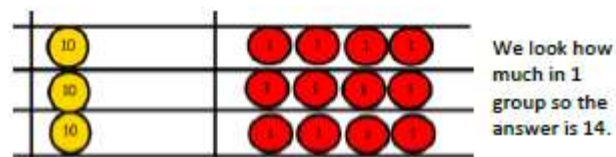
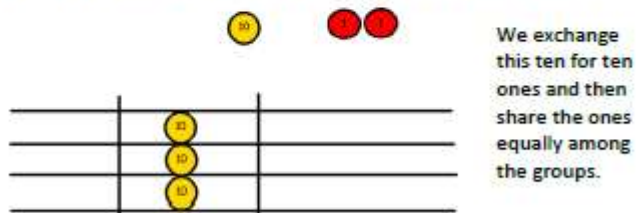
Short  
division  
(End of  
year 4+)



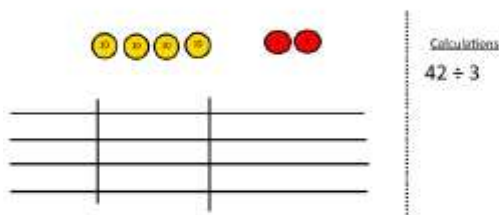
Use place value counters to divide using the bus stop method alongside



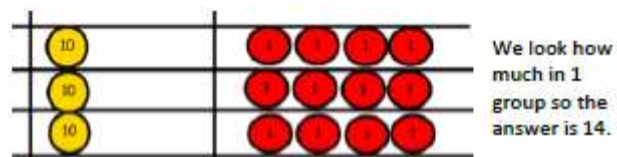
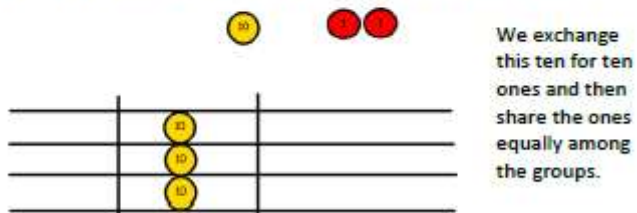
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



Use place value counters to divide using the bus stop method alongside



Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 4272} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$



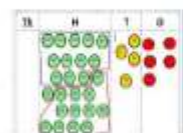


$$\begin{array}{r} 0212 \\ 12 \overline{)2544} \\ \underline{24} \phantom{00} \\ 14 \phantom{00} \\ \underline{12} \phantom{00} \\ 24 \phantom{00} \\ \underline{24} \phantom{00} \\ 0 \end{array}$$

$2544 \div 12$   
How many groups of  
12 thousands do we  
have? None



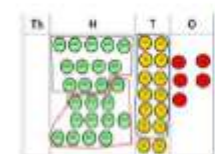
Exchange 2 thousand for  
20 hundreds.



$$\begin{array}{r} 02 \\ 12 \overline{)2544} \\ \underline{24} \phantom{00} \\ 1 \phantom{00} \end{array}$$

How many groups of  
12 are in 25  
hundreds? 2 groups.  
Circle them.

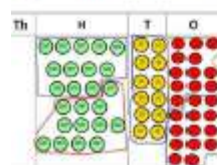
We have grouped 24 hundreds so can take  
them off and we are left with one.



$$\begin{array}{r} 021 \\ 12 \overline{)2544} \\ \underline{24} \phantom{00} \\ 14 \phantom{00} \\ \underline{12} \phantom{00} \\ 2 \phantom{00} \end{array}$$

Exchange the  
one hundred  
for ten tens so  
now we have 14  
tens. How many

groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for  
twenty ones so now we have  
24 ones. How many groups  
of 12 are in 24? 2

Children to represent the counters, pictorially  
and record the subtractions beneath.

$$\begin{array}{r} 0 \\ 12 \overline{)2544} \end{array}$$

Step one- exchange 2  
thousand for 20 hundreds  
so we now have 25  
hundreds.

$$\begin{array}{r} 02 \\ 12 \overline{)2544} \\ \underline{24} \phantom{00} \\ 1 \phantom{00} \end{array}$$

Step two- How many groups  
of 12 can I make with 25  
hundreds? The 24 shows the  
hundreds we have grouped.  
The one is how many  
hundreds we have left.

$$\begin{array}{r} 021 \\ 12 \overline{)2544} \\ \underline{24} \phantom{00} \\ 14 \phantom{00} \\ \underline{12} \phantom{00} \\ 2 \phantom{00} \end{array}$$

Exchange the one hundred  
for 10 tens. How many  
groups of 12 can I make  
with 14 tens?  
The 14 shows how many tens  
I have, the 12 is how many I

grouped and the 2 is how many tens I have  
left.

$$\begin{array}{r} 0212 \\ 12 \overline{)2544} \\ \underline{24} \phantom{00} \\ 14 \phantom{00} \\ \underline{12} \phantom{00} \\ 24 \phantom{00} \\ \underline{24} \phantom{00} \\ 0 \end{array}$$

Exchange the 2 tens for 20  
ones. The 24 is how many ones  
I have grouped and the 0 is  
what I have left.